ON THE ANALYSIS OF QUASI-PLANAR TRANSMISSION LINES IN CIRCULAR/ELLIPTICAL WAVEGUIDES USING THE METHOD OF LINES

By

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Abstract

The analysis of a class of quasi-planar transmission lines in circular/elliptical waveguides is introduced. Data will be presented to characterize wave propagation in microstrip and slotlines in closed and semi-open circular metallic enclosures. The method of lines has been modified for this problem to treat curved and open boundary value problems in this type of transmission lines.

INTRODUCTION

Since the early paper by Robertson [1] in 1955, describing an "Ultra-Bandwidth Finline Coupler", quasi-planar transmission lines in circular or elliptical waveguide housings have been almost totally neglected. Only recently a paper by Costache and Hoefer [2] gave some inside in dispersion characteristics of bilateral finlines in circular waveguides. On the other hand, quasi-planar transmission lines in circular/elliptical waveguides are of potential interest since they may provide a better control of field polarization for phase shifters, antenna feed lines, travelling wave isolators etc.

The purpose of this paper is to extend the knowledge about wave propagation for a larger variety of quasi-planar transmission lines shielded by circular/elliptical waveguide housing. In particular we will analyze microstrip, suspended microstrip, slotline and suspended slotline with closed circular or elliptic metallic shielding. We will also investigate the semi-open boundary case where the transmission line structure is enclosed by a half-circular shield and the groundplane removed. Fig. 1 shows a variety of structures to be investigated in this paper.

A rigorous analysis of wave propagation in bilateral finlines with circular housing using the Finite Element Method (FEM) was recently published in [2]. However, in this approach large systems of equations must be solved directly which requires considerable memory space and computing power. Using the Method Of Lines (MOL) instead we are able to apply a discrete orthogonal transformation and the eigenvalues problem is essentially solved

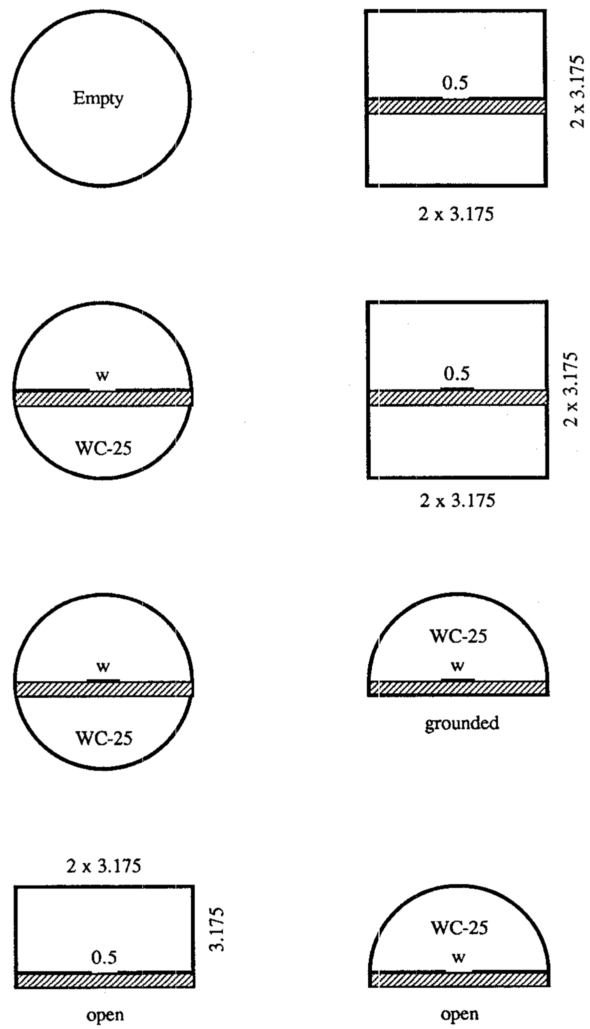


Figure 1:

Sample of quasi-planar transmission line cross-sections in rectangular, circular, semi-circular, closed and semi-closed metallic enclosures analytically in the transformed domain. This procedure requires only a one-dimensional discretization of the cross-section and thus reduces the matrix sizes in the original domain significantly. At the same time the computer algorithm is considerably faster and does not require supercomputer power.

In the past the MOL was mainly applied to simple and in most cases rectangular structures embedded in a metallic (rectangular) enclosure. In this paper we will modify the MOL to include also curved as well as semi-open boundaries.

THEORY

The principle steps involved can best be explained for the symmetrical stripline covered by a semi-circular shield, shown in Fig. 2. Consider first of all the MOL principle applied to the same structure with a rectangular shield. All field components in the individual subregions are found from a superposition of two scalar potentials

$$\vec{E} = \frac{1}{j\omega\epsilon}\nabla \times \nabla \times (\Psi^e e\vec{z}) - \nabla \times (\Psi^h e\vec{z}) \qquad (1)$$

$$\vec{H} = \nabla \times (\Psi^e e\vec{z}) + \frac{1}{j\omega\mu}\nabla \times \nabla \times (\Psi^h e\vec{z}) \qquad (2)$$

Discretizing the cross-section in x direction allows to write the x−dependent variables in vector form which includes the lateral boundary and edge conditions. Applying an orthogonal transformation and subsequently a decoupling procedure we are able to rewrite the partial differential Helmholtz equations as a system of ordinary differential equations which depend only on the y-direction. This leads to a set of inhomogeneous transmission line equations for each homogeneous substrate layer

$$\begin{pmatrix} [\phi] \\ \left[\frac{d\phi}{dy}\right] \end{pmatrix}_{y_2} = \begin{pmatrix} \left[\cosh\gamma^{e,h}(y_2-y_1)\right] & \left[\sinh\gamma^{e,h}(y_2-y_1)\right]\frac{1}{\gamma^{e,h}} \\ \left[\gamma^{e,h}\sinh\gamma^{e,h}(y_2-y_1)\right] & \left[\cosh\gamma^{e,h}(y_2-y_1)\right] \end{pmatrix} \begin{pmatrix} [\phi] \\ \left[\frac{d\phi}{dy}\right] \end{pmatrix}_{y_1} \qquad (3)$$

Up to here the method is well known from [3]-[5]. The matrix equation in (3) is a function of the y-coordinate at discrete points in x-direction and the boundary conditions are, once they are satisfied at one location in x, assumed to be satisfied at each x-coordinate along the boundary. This is not true for a curved waveguide housing. In this case the field components tangential (E_t) and normal (H_n) to the boundary are composed of vector components which vary along the x-coordinate. To account for this variation in boundary conditions in the discretization procedure we rewrite the function which describes the curvature of the waveguide housing

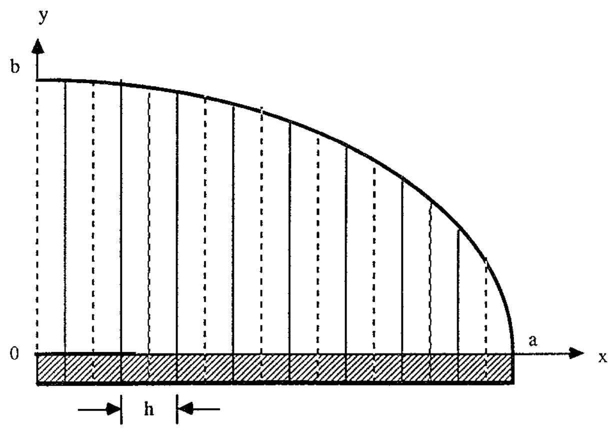


Figure 2:

Cross-section of a shielded microstrip transmission line in semi-elliptical metallic enclosure

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (4)$$

in terms of a tangential and a normal unity vector

$$\vec{U}_t = \frac{\vec{x} + \frac{d_y}{d_x}\vec{y}}{\sqrt{1+\left(\frac{d_y}{d_x}\right)^2}} \qquad (5)$$

$$\vec{U}_n = \frac{-\vec{x} + \left(\frac{d_x}{d_y}\right)\vec{y}}{\sqrt{1+\left(\frac{d_x}{d_y}\right)^2}} \qquad (6)$$

a and b in (4) denote the semi-axis lengths along the x- and y-directions, respectively. If we now apply the boundary conditions of zero tangential electric field and zero normal magnetic field in the original domain, we obtain two matrix equations with inner products at each discrete point

$$\left|\vec{U}_t\right| \bullet \left[\vec{E}_x + \vec{E}_y\right] = 0 \qquad (7)$$

$$\left|\vec{U}_n\right| \bullet \left[\vec{H}_x + \vec{H}_y\right] = 0 \qquad (8)$$

$$\left[\vec{E}_z\right] = 0 \qquad (9)$$

If we express the fields in (7)-(9) in terms of the potential functions transformed into the transform domain yields

$$[\phi^e] = 0 \qquad (10)$$

$$\left[\frac{d\phi^h}{d_y}\right] = \left[T^h\right]^t \left[\frac{d_y}{d_x}\right] \left[D^h\right] \left[T^h\right] \left[\phi^h\right] \qquad (11)$$

One can see that $[\phi^e]$ (proportional to E_z) remains always zero at the boundary and that the vector $\left[\frac{d\phi^h}{d_y}\right]$ depends on the x- y-coordinate along the boundary. Thus, all elements of the vector $\left[\frac{d\phi^h}{d_y}\right]$ are coupled to each other. Only in the limiting case of a piece-wise straight or rectangular boundary, $\left[\frac{d\phi^h}{d_y}\right]$ will become zero.

Eqn. (10) and (11) are directly linked to the interface condition at $y = 0$

$$\begin{pmatrix} [\tilde{E}_x] \\ \\ [\tilde{E}_z] \end{pmatrix} = \begin{pmatrix} [\tilde{Z}_{xx}] & [\tilde{Z}_{xz}] \\ \\ [\tilde{Z}_{zx}] & [\tilde{Z}_{zz}] \end{pmatrix} \begin{pmatrix} [\tilde{J}_x] \\ \\ [\tilde{J}_z] \end{pmatrix} \tag{12}$$

through the transmission line Eqn. (3) of the homogeneous subregion. Using the reverse transformation into the original domain [5] and applying the boundary condition on the conductor surface leads finally to the reduced characteristic matrix equation system

$$[Z(\beta)] \begin{pmatrix} [J_x] \\ \\ [J_z] \end{pmatrix} = 0 \tag{13}$$

which must be solved for the zeros of the determinant to provide the propagation constant of the fundamental and higher order modes.

RESULTS

Fig. 3 shows the dispersion diagram of the fundamental mode for two different standard circular waveguides. The numerical results agree perfectly with those obtained analytically. In Fig. 4 we compare the propagation characteristics of a unilateral finline in a square waveguide housing with that in a circular waveguide housing (WC-25). For the electric wall symmetry the monomode range in the WC-25 waveguide by far exceeds that of the square waveguide. For magnetic wall symmetry no modes were found in the frequency range of interest. Fig. 5 shows a comparison between a suspended microstrip structure and its grounded counterpart. As expected, the propagation constant for the grounded microstrip structure is much higher than that for the suspended one. Increasing the strip widths in the grounded stripline leads to a higher propagation constant, while in the case of the circular shielding the propagation constant decreases. Both effects are already known from the shielded microstrip line with rectangular enclosure and its suspended counterpart. Mode dispersion in an open bottom slotline covered with a rectangular metallic shield is shown in Fig. 6. For comparison a semi-circular (open bottom) slotline is shown in Fig. 7. In contrast to the ew symmetry of the structure in Fig. 7, where only one mode can exist (three modes in the rectangular wg), the magnetic wall symmetry (mw), shows the existence of two fundamental modes. In other words, depending on the polarization of the incident TE_{11}-mode in the circular quasi-planar structure, one can exploit a relatively large single mode range for this semi-open transmission line.

CONCLUSION

This paper presents for the first time, a detailed analysis of a variety of quasi-planar transmission lines in circular/elliptical waveguide housings. Using the Method Of Lines, which has been modified to treat curved and open boundary value problems, we have presented numerical results describing wave propagation in shielded and semi-open slotline and microstrip lines.

REFERENCES

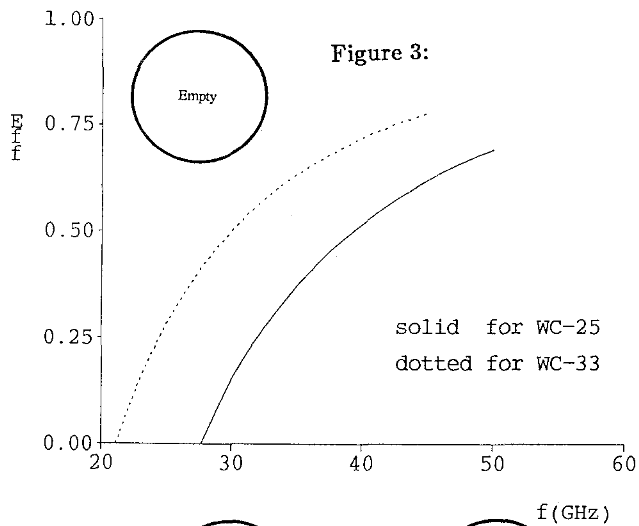
[1] Robertson, S.D., "The Ultra-Bandwidth Finline Coupler", Proc. IRE, vol. 43, no. 6, pp. 739-741, June 1985.

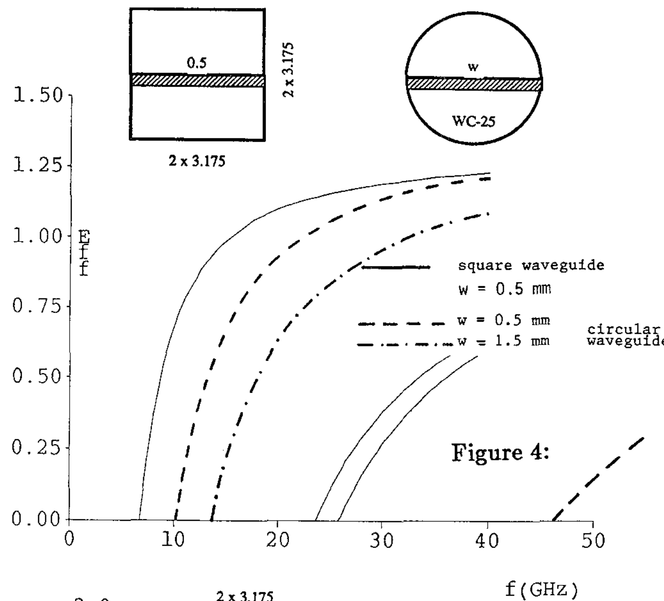
[2] Eswarappa, G. Costache and W.J.R. Hoefer, "Finlines in Rectangular and Circular Waveguide Housings Including Substrate Mounting and Bending Effects-Finite Element Analysis", IEEE Trans. MTT-37, pp. 299-306, Feb. 1989.

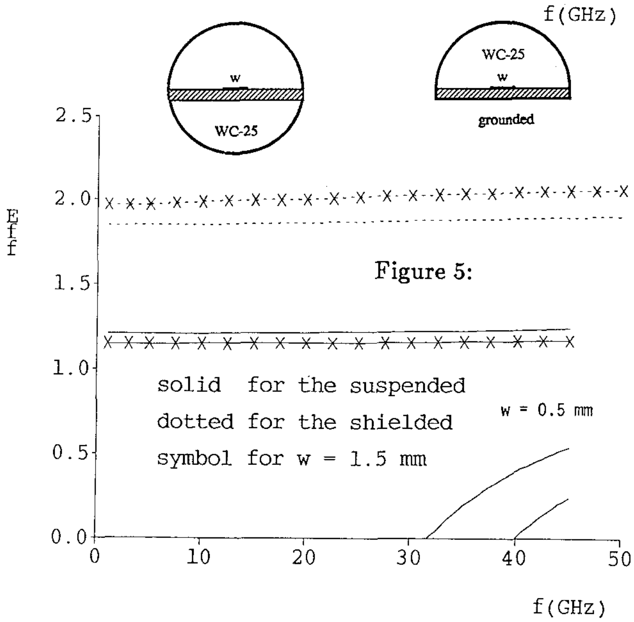
[3] Schulz, U., R. Pregla, "A New Technique for the Analysis of the Dispersion Characteristics of Planar Waveguides", AEÜ 34, pp. 169-173, 1980.

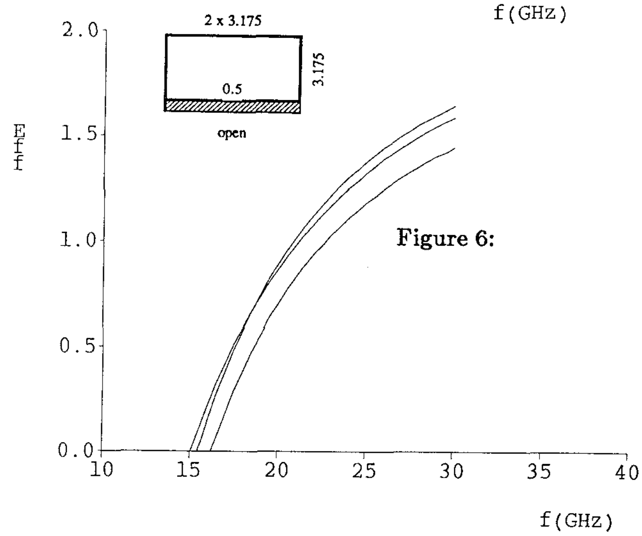
[4] Worm, S. and R. Pregla, "Hybrid-mode Analysis of Arbitrarily Shaped Planar Microwave Structures by the Method of Lines", IEEE Trans. Microwave Theory Tech., vol. MTT-32, pp. 191-196, Feb. 1986.

[5] Pregla, R. and S. Worm, "The Method of Lines for the Analysis of Planar Waveguides with Magnetized Ferrite Substrates", in IEEE MTT-S 1984 Int. Microwave Symp. Dig., pp. 348-350, May 1984.









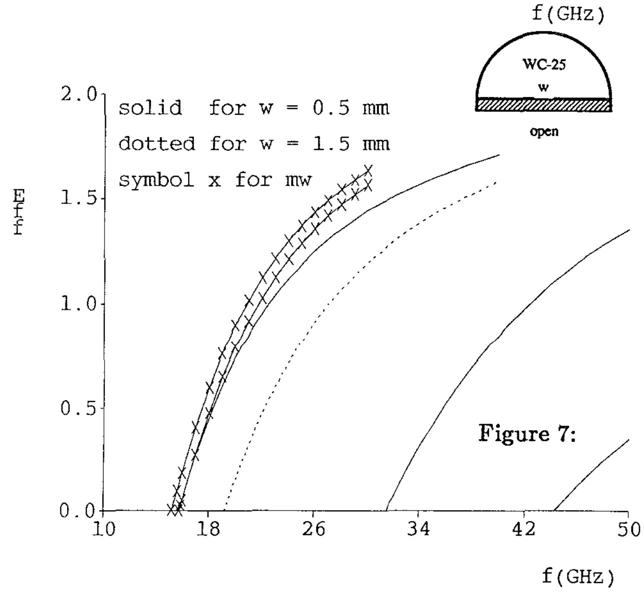


Figure 3:	Dispersion diagram of the standard circular waveguide
Figure 4:	Finline dispersion in a square and circular waveguide
Figure 5:	Stripline dispersion in circular and semi-circular waveguide
Figure 6:	Open-bottom slotline dispersion in a rectangular shielding
Figure 7:	Open-bottom slotline dispersion in a semi-circular shielding